JIE Summer School

Lecture 3B: Accounting for the Margins

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I. Extensive, Price, and Quantity Margins

based on Eaton and Cecília Fieler "The Margins of Trade" (2022)

Standard Elements

- N countries indexed by destination n and source i
- An endogenous measure of varieties indexed by $\omega \in \Omega$
- Monopolistic competition with heterogeneous firms
- Worker-households in exogenous measures L_i
- Mobile across ω within i.

Unusual Element: Two Dimensions of Quality

- “horizontal” quality valued equally by all
  - standard in models explaining why rich countries sell more expensive goods
  - consistent with homotheticity
  - substitutes for quantity
- “vertical” quality, a luxury
  - standard in models explaining why rich countries buy more expensive goods
  - introduces nonhomotheticity
  - complementary with quantity

Demand

- Aggregate Y from a continuum of varieties (used by households for consumption or by firms as intermediates)

$$Y = \left[\int_{\omega\in\Omega} u(\omega)^{\beta} d\omega\right]^{1/\beta}$$
$$u(\omega) = \left[(Q(\omega)y(\omega))^{\rho} + q(\omega)^{\rho}\right]^{1/\rho}$$

  where $y(\omega)$ is the quantity of variety ω

- The two dimensions of quality:
  - $Q(\omega)$, "horizontal quality", substitutes for quantity
  - $q(\omega)$ "vertical quality", complementary with quantity
- $\beta \leq 1$ and $\rho < 0$
- Like Bekkers, Francois, and Manchin (2012) with Q added

Examples of two-dimensional quality

- Parts of a product (e.g., hubs and spokes of cycles)
  - Goods with a low Q may break in the assembly or not have the correct dimensions
  - q may improve the performance of the final good
- Clothing (e.g., baby clothing)
  - Q is durability, warmth
  - q is stylishness

Technology

- Constant returns to scale
- A worker at firm ω making product ω can make:

$$
\begin{aligned}
y(\omega) &= z(\omega)m(\omega)^{1-\alpha}q(\omega)^{-\gamma} \\
Q(\omega) &= z(\omega)^{\eta}m(\omega)^{\nu}
\end{aligned}
$$

where

- $z(\omega)$ efficiency of firm ω
- $m(\omega)$ amount of aggregate Y used as intermediates per worker
- γ sacrifice of efficiency to achieve greater $q(\omega)$
- 1 - α contribution of intermediates to $y(\omega)$ given $q(\omega)$
- η contribution of $z(\omega)$ to $Q(\omega)$
- ν contribution of $m(\omega)$ to $Q(\omega)$

To Solve

- **The Buyer's Problem:** Given a budget X and the price $p(\omega')$, horizontal quality $Q(\omega')$, and vertical quality $q(\omega')$ of each available variety ω', the buyer chooses each $y(\omega)$ to maximize Y

- **The Producer's Problem:** The producer chooses $p(\omega)$, $y(\omega)$, $q(\omega)$, $Q(\omega)$, $m(\omega)$, and labor $l(\omega)$ to maximize profit given the buyer's first-order condition for choosing $y(\omega)$ (from above), and given the wage w and cost of inputs $X\left(m(\omega)\right)$, where the cost function $X(\cdot)$ is derived below.

Unit costs and cost index

- Denote:
  - firm ω's cost to produce one unit of $y(\omega)$, with $q(\omega) = 1$, given $Q(\omega)$ and $m(\omega)$, as $c(\omega)$
  - firm ω's inverse horizontal-quality adjusted unit cost as

$$v(\omega) = \frac{Q(\omega)}{c(\omega)},$$

  - the markup

$$\overline{m} = (1 + \gamma)/\beta$$

    (instead of the standard $1/\beta$)

Expenditure function

- The inverse horizontal-quality adjusted unit cost index:

$$V = \left[\int_{\omega \in \Omega} v(\omega)^{1/(\bar{m}-1)} d\omega \right]^{\bar{m}-1}.$$

- The budget X needed to purchase Y is then:[1]

$$X(Y) = \Gamma_3 Y^{1+\gamma} V^{-1}$$

- A buyer with budget X facing a price index V spends

$$x(\omega) = p(\omega) y(\omega) = \left(\frac{v(\omega)}{V} \right)^{1/(\overline{m}-1)} X,$$

on product ω with inverse unit cost $v(\omega)$

[1] Here and below $\Gamma_k; k = 1, 2, \ldots$ are uninteresting constants that depend on parameters β, γ, ρ, ...

Qualities, prices, and costs

- (suppressing ω)

$$
\begin{aligned}
q &= \Gamma_1^{1/\rho} Q y \\
Q &= \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}} \frac{wV}{\Gamma_3} \right)^{\nu/(1+\gamma)} z^{\eta} \\
p &= \overline{m} c q^{\gamma}
\end{aligned}
$$

  where

$$
\tilde{\alpha} = \frac{\alpha + \gamma - \nu}{1+\gamma}.
$$

  is the labor share

- horizontal-quality-adjusted unit cost

$$
\tilde{c} = \frac{z^{1+\eta}}{Q} c = \tilde{\alpha}^{-\tilde{\alpha}} (1-\tilde{\alpha})^{-(1-\tilde{\alpha})} w^{\tilde{\alpha}} (\Gamma_3 V^{-1})^{1-\tilde{\alpha}}
$$

Introducing Geography

- Source i has a measure of potential producers $T_i z^{-\theta}$ with efficiency $Z \geq z$
- Entry into destination n costs $f_n = \kappa_0 \tilde{c}_n L_n^{1+\kappa_1}$
- Iceberg trade costs $d_{ni} \geq 1$ to destination n from source i
- Expenditure X_n in destination n

Entry

- The inverse quality-adjusted unit cost of a seller from source i with efficiency z in destination n

$$v_{ni}(z) = \frac{z^{1+\eta}}{d_{ni}\tilde{c}_i}$$

- the zero-profit condition implies the minimum $v_{ni}(z)$ for entry

$$\underline{v}_n = \Gamma_5 \left(\frac{f_n}{X_n} \right)^{\bar{m}-1} V_n$$

Isolating pure randomness

- Define:

$$\epsilon_{ni}(\omega) = v_{ni}(z(\omega)) / \underline{v}_n$$

which is distributed Pareto:

$$Pr[\epsilon_{ni} \leq \epsilon] = 1 - \epsilon^{-\tilde{\theta}}$$

where:

$$\tilde{\theta} = \frac{\theta}{1 + \eta}$$

- so is independent of *n* or *i* (pure randomness)

Price index and trade share

- Price term:

$$V_n = \Gamma_7 \left(\frac{X_n}{f_n} \right)^{\bar{m}-1-1/\tilde{\theta}} \Phi_n^{1/\tilde{\theta}}$$

$$\Phi_n = \sum_{i=1}^{N} T_i (d_{ni} \tilde{c}_i)^{-\tilde{\theta}}$$

- Trade share:

$$\pi_{ni} = \frac{T_i (d_{ni} \tilde{c}_i)^{-\tilde{\theta}}}{\Phi_n}$$

Bilateral price

$$p_{ni}(\epsilon, x) = \Gamma_8 \underbrace{\left(d_{ni} w_i^{\tilde{\alpha}} V_i^{-(1-\tilde{\alpha})}\right)}_{\text{cost}} \underbrace{\left(d_{ni} w_i^{\tilde{\alpha}} V_i^{-(1-\tilde{\alpha})}\right)^{\tilde{\eta}-1} \left[\left(\frac{f_n}{X_n}\right)^{\overline{m}-1} V_n\right]^{\tilde{\eta}-1}}_{\text{selection}}$$

$$\underbrace{(w_i V_i)^{\nu(1-\tilde{\gamma})}}_{\text{horizontal quality}} \underbrace{\left(\frac{f_n}{X_n}\right)^{\tilde{\gamma}\bar{m}}}_{\text{competition}} \underbrace{(xV_n)^{\tilde{\gamma}}}_{\text{non-homothetic demand}} \epsilon^{\tilde{\eta}-1+\tilde{\gamma}\overline{m}/(\overline{m}-1)}$$

where $\tilde{\eta} = \eta/(1+\eta)$; $\tilde{\gamma} = \gamma/(1+\gamma)$

- Melitz case: $\nu = \eta = 0$ and $\gamma \to 0$.
- If $\eta > 0$ quality-adjusted cost decreases with z at a rate $\eta + 1$
- If $\nu > 0$ higher wage and cheaper inputs increase horizontal Q.
- If $\gamma > 0$ vertical quality (and price) rise with spending per buyer

The extensive margin

- A unit continuum of products indexed by k. The probability that a variety is in product with index less than k is

$$F(k) = k^{\kappa_2}$$

where $\kappa_2 > 1$.

- The number of varieties from i in product k with efficiency $Z \geq z$ is distributed Poisson with parameter:

$$dF(k) T_i z^{-\theta}.$$

- Giving us predictions for the the number of products that n imports from i, E_{ni}, that n imports, $E_{n\cdot}$, and that i exports, $E_{\cdot i}$

Estimation 1: Gravity

- Regress

$$\log\left(\frac{\pi_{ni}}{\pi_{nn}}\right) = A_n + B_i + \delta^g \log \text{dist}_{ni} + \epsilon^g_{ni}$$

where

$$\pi_{ni} = \frac{X_{ni}}{X_n}$$
$$X_n = \frac{w_n L_n}{\tilde{\alpha}} + D_n$$

D_n is the deficit and we allow $d_{nn} \neq 1$ and fix $\tilde{\alpha} = 0.5$

- to recover:

$$\widehat{d_{ni}^{-\tilde{\theta}}} = \hat{\delta^g} \log \text{dist}_{ni}$$
$$\widehat{\Phi}_n = \exp(-\hat{A}_n) + \sum_{i \neq n} \exp(\hat{B}_i + \hat{\delta^g} \log \text{dist}_{ni})$$

Estimation 2: Decomposition into Margins

- Parameters $\Xi = \{\gamma, \eta, \nu, \theta, \beta, \kappa_0, \kappa_1, \kappa_2\}$ to minimize:

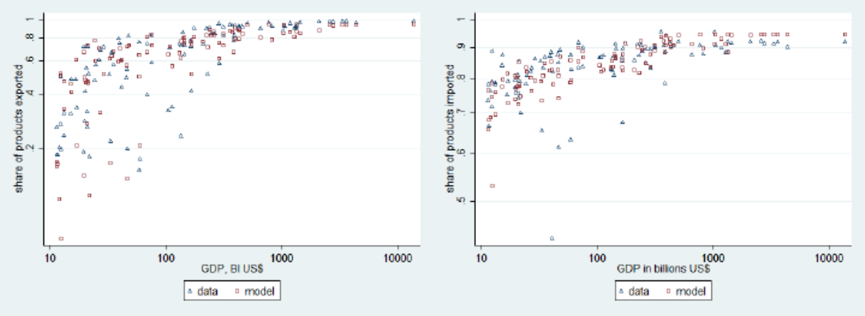
$$
\begin{aligned}
\mathcal{W}(\Xi) = & \frac{1}{N_P V(\log \overline{P}_{ni}^{\text{data}})} \sum_{n=1}^{N} \sum_{i \neq n, i=1}^{N} \left(\log \overline{P}_{ni}^{\text{model}}(\Xi) - \log \overline{P}_{ni}^{\text{data}} \right)^2 \\
& + \frac{1}{N_E V(\log E_{ni}^{\text{data}})} \sum_{n=1}^{N} \sum_{i \neq n, i=1}^{N} \left(\log E_{ni}^{\text{model}}(\Xi) - \log E_{ni}^{\text{data}} \right)^2 \\
& + \frac{1}{N V(\log E_{\cdot i}^{\text{data}})} \sum_{i=1}^{N} \left(\log E_{\cdot i}^{\text{model}}(\Xi) - \log E_{\cdot i}^{\text{data}} \right)^2 \\
& + \frac{1}{N V(\log E_{n \cdot}^{\text{data}})} \sum_{n=1}^{N} \left(\log E_{n \cdot}^{\text{model}}(\Xi) - \log E_{n \cdot}^{\text{data}} \right)^2 .
\end{aligned}
$$

Parameter estimates

	parameter estimate	standard error
γ	0.156	0.040
η	0.352	0.123
ν	0.093	0.025
θ	7.758	1.905
β	0.563	0.012
κ_1	-0.425	0.008
κ_2	5.103	0.204
κ_0	0.770	0.118

	number of observations	R-squared
$\log \overline{P}_{ni}$	9479	0.53
$\log E_{ni}$	9558	0.38
$\log E_{n\cdot}$	100	0.19
$\log E_{\cdot i}$	100	0.65

Extensive margins model x data



(a) EM of exporting (b) EM of importing

Margins of trade

	data				model			
	value	EM	quantity	price	value	EM	quantity	price
exporter GDP	1.36	0.88	0.45	0.03	1.37	0.84	0.46	0.06
importer GDP	1.11	0.40	0.66	0.05	1.01	0.49	0.58	0.04
distance	-1.19	-0.72	-0.51	0.04	-1.18	-0.65	-0.56	0.02
exporter GDP per capita	1.35	0.92	0.33	0.10	1.36	0.85	0.33	0.17
exporter population	1.36	0.85	0.55	-0.03	1.37	0.83	0.56	-0.03
importer GDP per capita	1.09	0.46	0.51	0.13	0.97	0.52	0.46	0.11
importer population	1.13	0.35	0.80	-0.02	1.05	0.47	0.68	-0.01
distance	-1.20	-0.68	-0.62	0.10	-1.20	-0.63	-0.65	0.09
number of observations	9,479	9,479	9,479	9,479	9,479	9,479	9,479	9,479

The model's decomposition of values into margins has 8 parameters.

Distribution of the number of exporters per product

	percentile of the distribution					
	10%	25%	50%	75%	90%	**mean**
data	35	51	68	81	91	**65**
model	13	47	78	90	94	**66**

The Gains from Trade

- Welfare U_n proportional to $W_n V_n$
- The ACR formula

$$U_n = \Gamma_{12} \left(L_n^{\kappa_1 \left[1 - \tilde{\theta}(\overline{m} - 1)\right]} \cdot \frac{T_n d_{nn}^{-\tilde{\theta}}}{\pi_{nn}} \right)^{\varsigma_1 / \tilde{\theta}}$$

Decomposing Gains

- The gains from a greater range of varieties U_n^R

$$\hat{U}_n^R = \hat{U}_n^{(1-\tilde{\alpha})(\bar{m}-1)} \hat{L}_n^{-\kappa_1(\bar{m}-1)}.$$

- Our estimates put the coefficient on $\hat{U}_n$ at 0.53
- Gains from more people, $\hat{L}_n$, come more in the form of greater range of varieties
- But, aggregating to the level of HS6 *products*, much less of the gains appear as greater range and more as lower cost

II. Buyer and Seller Margins

based on Eaton, Sam Kortum, and Francis Kramarz "Firm-to-Firm Trade: Imports, Exports, and the Labor Market" (2022)

Basic Elements

- $\mathcal{N}$ countries indexed by destination n and source i
- Country i has L_i^l workers of type l

Producers

- producer j in i has efficiency $z(j)$
- K types of tasks each with Cobb-Douglas share $\beta_{k,i}$
- a task of type k can be performed by an appropriate intermediate or by the type of labor appropriate for that type of task $l(k)$
- elasticity of substitution σ between tasks of a given type

Unit Costs

Firm j in source i has unit cost in destination n:

$$c_{ni}(j) = \delta_n(j)\bar{c}_{ni}(j) = \delta_n(j)\frac{d_{ni}C_i(j)}{z(j)}$$

where:

- $\bar{c}_{ni}(j)$ is j's *core cost* in destination n
- $\delta_n(j)$ is j's *idiosyncratic cost* in destination n
- core cost is input cost $C_i(j)$ times iceberg cost d_{ni} divided by efficiency $z(j)$

Tasks

- $K+1$ types of tasks, each with Cobb-Douglas share $\beta_{k,i}$
- For producer j each type involves $m(j)$ tasks with elasticity of substitution σ between them.
- producer j's cost of performing task ω is $c_{k,i}(j,\omega)$, which can differ across producers for the same task.
- producer j's input cost is thus

$$C_i(j) = g_i(m(j)) \prod_{k=0}^{K} \left(\left(\sum_{\omega=1}^{m(j)} c_{k,i}(j,\omega)^{-(\sigma-1)} \right)^{-1/(\sigma-1)} \right)^{\beta_{k,i}}$$

(where $g_i(m)$ kills the love-of-variety effect on unit cost).

Performing Tasks with Labor or Intermediates

- Performing task ω of type k with labor requires $a_k(j,\omega)$ workers of the appropriate type $l(k)$ with wage $w_{k,i} = w^{l(k)}$
- The cheapest available intermediate to producer j for task ω of type k costs $\tilde{c}_{k,i}(j,\omega)$
- Hence its cost to perform the task is

$$c_{k,i}(j,\omega) = \min\left\{a_k(j,\omega)w_{k,i}, \tilde{c}_{k,i}(j,\omega)\right\}$$

Distributional Assumptions

- Measure of potential producers in country i with efficiency $z(j) > z$ with m tasks of each type

$$\mu_i^Z(z; m) = \frac{p(m)}{g_i(m)} T_i z^{-\theta}$$

- $F(a)$: distribution of $a_k(j, \omega)$

Retailers

- Same production structure as producers
- Buy from domestic and foreign producers
- Sell an aggregate of manufactures to local households and the local service sector
- Common efficiency $z = 1$
- Exogenous measure F_i^R

Presence of Buyers

- Total measure of firms: $F_n = F_n^P + F_n^R$ where F_n^P is endogenous (determined below)
- Average number of tasks per type: $\overline{m}$
- Buyer presence $B_n = \overline{m} F_n$

Presence of Sellers

- Measure of sellers in n from i with cost below c: $\mu_{ni}(c)$ (derived below)
- Seller presence

$$S_n(c) = \sum_i \lambda_{ni} \mu_{ni}(c)$$

Buyer-Seller Matching

- A seller with unit cost c meets a buyer for a task of type k with intensity:

$$\lambda_{k,ni}(c) = \lambda_k \lambda_{ni} B_n^{-\varphi} S_n(c)^{-\gamma}$$

- φ and γ reflect congestion in matching from buyers and sellers
- λ_k reflects matching intensity across types of tasks (with $\sum_k \lambda_k = 1$)
- λ_{ni} reflects matching intensity between different pairs of countries

Number of Matches

For a seller in i with unit cost *exactly* c the number of matches for a task of type k with a buyer in n is distributed Poisson with parameter

$$e_{k,ni}(c) = \lambda_{k,ni}(c) B_n = \lambda_k \lambda_{ni} B_n^{1-\varphi} S_n(c)^{-\gamma}$$

Measure of Matches

- The measure of matches between buyers in n and sellers from country i with price (=unit cost) *below* c for tasks of type k is:

$$M_{k,ni}(c) = \sum_i \int_0^c e_{k,ni}(c')d\mu_{ni}(c') = \frac{1}{1-\gamma}\lambda_k \lambda_{ni} B_n^{1-\varphi} \mu_{ni}(c) S_n(c)^{-\gamma}$$

- The measure of matches between buyers in n and sellers from *anywhere* with price below c for tasks of type k is:

$$M_{k,n}(c) = \sum_i M_{k,ni}(c) = \frac{1}{1-\gamma}\lambda_k B_n^{1-\varphi} S_n(c)^{1-\gamma}$$

- The measure of matches between buyers in n and sellers from anywhere with price below c for *any* task is:

$$M_n(c) = \sum_k M_{k,n}(c) = \frac{1}{1-\gamma} B_n^{1-\varphi} S_n(c)^{1-\gamma}$$

Number of Quotes

- The number of *quotes* below price c that u buyer in n receives for a task of type k from sellers from i is distributed Poisson with parameter:

$$\rho_{k,ni}(c) = \frac{M_{k,ni}(c)}{B_n} = \frac{\lambda_k}{1-\gamma}\lambda_{ni}\mu_{ni}(c)B_n^{-\varphi}S_n(c)^{-\gamma}.$$

- Aggregating across potential suppliers from each source i, the number of quotes from anywhere with cost is distributed Poisson with parameter:

$$\rho_{k,n}(c) = \frac{M_{k,n}(c)}{B_n} = \frac{\lambda_k}{1-\gamma}B_n^{-\varphi}S_n(c)^{1-\gamma}$$

The Distribution of the Lowest Cost

- Evaluating the Poisson distribution at zero, the probability that a buyer encounters no supplier with unit cost below c is $e^{-\rho_{k,n}(c)}$.
- A buyer can also perform task ω with labor at unit cost $a_k(j, \omega) w_{k,n}$, which exceeds c with probability $1 - F(c / w_{k,n})$.
- Since the two events are independent, the distribution of the lowest cost to fulfill such a task is:

$$G_{k,n}(c) = 1 - e^{-\rho_{k,n}(c)} [1 - F(c / w_k)]$$

Home Suppliers I

- Measure of potential producers in i with core cost below $\bar{c}$ at home: $\bar{\mu}_{ii}(\bar{c})$
- Conditional on input cost C_i, the measure with core cost below $\bar{c}$ at home:

$$\bar{\mu}_{ii}(\bar{c}|C_i) = \mu_i^Z \left(\frac{C_i}{\bar{c}}\right) = T_i C_i^{-\theta} \bar{c}^{\theta}.$$

Home Suppliers II

- Integrating over the components of C_i using $G_{k,n}(c)$, and summing over the distribution of m, the measure of potential producers from i with core cost below $\bar{c}$ at home:

$$\bar{\mu}_{ii}(\bar{c}) = T_i \Xi_i \bar{c}^{\theta}$$

- where:

$$\Xi_i = \sum_m \frac{p(m)}{g(m)^{\theta}} \prod_k \int_0^{\infty} \dots \int_0^{\infty} \left(\sum_{\omega=1}^{m} c_{\omega}^{-(\sigma-1)} \right)^{\theta\beta_{k,i}/(\sigma-1)} dG_{k,i}(c_1)\dots dG_{k,i}(c_m)$$

Suppliers to Destination n

- Measure of suppliers to n from i with unit cost below c

$$\mu_{ni}(c) = \int \bar{\mu}_{ii}\left(c/\left(d_{ni}\delta\right)\right) dG(\delta) = d_{ni}^{-\theta} T_i \Xi_i c^{\theta},$$

  normalizing:

$$\int \delta^{-\theta} dG(\delta) = 1.$$

- Measure of suppliers to n with unit cost below c

$$S_n(c) = \Upsilon_n c^{\theta} \tag{1}$$

  where

$$\Upsilon_n = \sum_i \lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i.$$

Number of Quotes and Labor Efficiency

- Number of quotes with unit cost less than c for task k (from above) is distributed Poisson with parameter:

$$\begin{aligned}\rho_{k,n}(c) &= \frac{\lambda_k}{1-\gamma} B_n^{-\varphi} S_n(c)^{1-\gamma} \\ &= \frac{\lambda_k}{1-\gamma} B_n^{-\varphi} \Upsilon_n^{1-\gamma} c^{\theta(1-\gamma)}.\end{aligned}$$

- Assume a distribution of labor efficiency to perform any task ω as:

$$F(a) = 1 - \exp\left(-a^{\theta(1-\gamma)}\right).$$

Solving the Cost Distribution

- Distribution of the lowest cost to fulfill task of type k in destination n:

$$G_{k,n}(c) = 1 - \exp\left(-\Phi_{k,n} c^{\theta(1-\gamma)}\right), \tag{2}$$

- where:

$$\Phi_{k,n} = \frac{\lambda_k}{1-\gamma} B_n^{-\varphi} \Upsilon_n^{1-\gamma} + w_{k,n}^{-\theta(1-\gamma)},$$

- which we can use to solve Ξ_i to get:

$$\Xi_i = \prod_k \Phi_{k,i}^{\beta_{k,i}/(1-\gamma)}.$$

Solving for Υ's

- Installing Ξ_i into Υ give the system of equations:

$$\Upsilon_n = \sum_i \lambda_{ni} d_{ni}^{-\theta} T_i \prod_k \left(\frac{\lambda_k}{1-\gamma} B_i^{-\varphi} \Upsilon_i^{1-\gamma} + w_{k,i}^{-\theta(1-\gamma)} \right)^{\beta_{k,i}/(1-\gamma)},$$

- The solution, given B and w, delivers the Υ's.
- Feed the Υ's into the Φ's to get the Ξ's
- To guarantee a unique solution for Υ, restrict $\lambda_0 = 0$ (with $\beta_{0,i} > 0$) to make sure that labor is always required.

Number of Buyers per Seller

- The number of buyers for a task of type k for a producer from i in n is distributed Poisson with parameter:

$$\eta_{k,ni}(c) = e_{k,ni}(c)(1 - G_{k,n}(c)) = e_{k,ni}(c) \exp\left(-\Phi_{k,n} c^{\theta(1-\gamma)}\right).$$

- Summing across k, this producer's number of buyers in market n is distributed Poisson with parameter:

$$\begin{aligned} \eta_{ni}(c) &= \sum_k \eta_{k,ni}(c) \\ &= \lambda_{ni} B_n^{1-\varphi} \Upsilon_n^{-\gamma} c^{-\theta\gamma} \sum_k \lambda_k \exp\left(-\Phi_{k,n} c^{\theta(1-\gamma)}\right) \end{aligned}$$

Measure of Buyers

- The number of buyers anywhere for a producer with unit cost c at home is distributed Poisson with parameter

$$\eta_i(c) = \sum_{n=1}^{\mathcal{N}} \eta_{ni}(cd_{ni}).$$

- The probability that the producer has at least 1 buyer is $1 - e^{-\eta_i(c)}$. The measure of active producers in i is thus:

$$F_i^P = \int_0^\infty \left(1 - e^{-\eta_i(c)}\right) d\mu_{ii}(c).$$

- Adding in the exogenous measure of retailers gives $F_i = F_i^P + F_i^R$, delivering the measure of buyers B_i.

Labor Shares

The probability that labor performs task ω of type k

$$1 - \varpi_{k,n} = \frac{w_{k,n}^{-\theta(1-\gamma)}}{\Phi_{k,n}}$$

Trade Shares

The probability that a good in n comes from i

$$\pi_{ni} = \frac{\rho_{k,ni}(c)}{\rho_{k,n}(c)} = \frac{\lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i}{\Upsilon_n} = \frac{\lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i}{\sum_{i'} \lambda_{ni'} d_{ni'}^{-\theta} T_{i'} \Xi_{i'}}.$$

regardless of k

Labor-Market Equilibrium

- GDP is:

$$Y_n = \sum_l w_n^l L_n^l.$$

- Final spending X_n^F is GDP plus the overall deficit $D_n = D_n^G + D_n^S$.
- Final spending on goods is $\alpha_n^G X_n^F$ and on services $\alpha_n^S X_n^F$.
- Output of producers in country i:

$$Y_i^P = \sum_n \pi_{ni} X_n^P$$

- Spending on labor of type l in country i is:

$$w_i^l L_i^l = \beta_i^{G,l} Y_i^G + \beta_i^{S,l} Y_i^S$$

where Y_i^G is output of the goods sector, including retail, and Y_i^S is output of services.

The Gains from Trade

Welfare:

$$U_i = \left(w_i^{\theta} \Xi_i \right)^{(\alpha_i^G + \alpha_i^S \beta_i^{SG})/\theta},$$

which solves:

$$U_i = \prod_{k \geq 1} \left(\frac{\lambda_k}{1-\gamma} O_i U_i^{\theta(1-\gamma)/(\alpha_i^G + \alpha_i^S \beta_i^{SG})} + 1 \right)^{\beta_{k,i}(\alpha_i^G + \alpha_i^S \beta_i^{SG})/[\theta(1-\gamma)(1-\beta_i^{SG}\beta_i^{GS})]},$$

with:

$$O_i = B_i^{-\varphi} \left(\frac{\lambda_{ii} T_i}{\pi_{ii}} \right)^{1-\gamma}.$$

Implications for Observables I

- Relationships

$$R_{ni} = \int_0^\infty \eta_{ni}(c) d\mu_{ni}(c) = \pi_{ni} \overline{\varpi}_n B_n$$

- Number of sellers

$$N_{ni} = d_{ni}^{-\theta} \int_0^\infty (1 - e^{-\eta_{ni}(c)}) d\mu_{ii}(c).$$

Implications for Observables II

- Buyers per Seller

$$\bar{b}_{ni} = \frac{R_{ni}}{N_{ni}} = \frac{\overline{\varpi}_n B_n^{1-\varphi/(1-\gamma)} \lambda_{ni}}{\int_0^\infty (1 - e^{-\eta_{ni}(c)}) d\mu_{ni}(c)}$$

- which increases in λ_{ni} but d_{ni} doesn't appear!
- Hence relationships relate to π_{ni} which reflects $d_{ni}^{-\theta}\lambda_{ni}$ while buyers per seller reflects only λ_{ni}

Gravity: Icebergs or Matching Frictions?

- the trade share

$$\pi_{ni} = \frac{\lambda_{ni} d_{ni} - \theta T_i \Xi_i}{Y_n}$$

falls with distance with an elasticity around -1.69 (on the large side)

- We find that -1.03 is due to lower matching frictions and -0.63 is due to higher iceberg trade costs
- This breakdown is informed by the effect of distance on buyers per seller relative to market share.

Parting Thoughts

- International economists (trade and finance) now have access to a vast array of data.
- These data exhibit some remarkable and surprising regularities.
- We should uncover and exploit these regularities to impose discipline on our how we model the international economy.